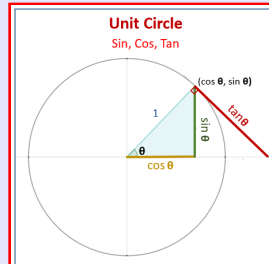


Trigonometry

Lecture 38



Feb 19-8:47 AM

More equations:

Solve $\sin 2x - \cos x = 0$ on $[0, 2\pi)$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

Use Zero-Product Rule

$$\cos x = 0 \quad 2 \sin x - 1 = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$$\frac{\pi}{2}, \frac{3\pi}{2} \quad \text{QI, QII}$$

$$\text{RA } \frac{\pi}{6}$$

$$\left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$$

$$\text{QI: } x = \frac{\pi}{6}$$

$$\text{QII: } x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Nov 6-10:29 AM

Solve $\tan \frac{x}{2} - \sin x = 0$ on $[0^\circ, 360^\circ)$.

Recall

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$= \frac{\sin A}{1 + \cos A}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\{0^\circ, 90^\circ, 270^\circ\}$$

$$\frac{1 - \cos x}{\sin x} - \sin x = 0$$

Multiply by $\sin x$

$$1 - \cos x - \sin^2 x = 0$$

$$1 - \cos x - (1 - \cos^2 x) = 0$$

$$\cancel{1} - \cos x - \cancel{1} + \cos^2 x = 0$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0$$

$$90^\circ, 270^\circ$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0^\circ$$

Nov 6-10:34 AM

Solve

$$\cos 7x - \cos 5x = 0$$

$$-2 \sin \frac{7x+5x}{2} \sin \frac{7x-5x}{2} = 0$$

Recall

$$\cos A - \cos B =$$

$$-2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$-2 \sin 6x \sin x = 0$$

$$\sin 6x = 0$$

$$6x = 0^\circ + K \cdot 360^\circ$$

$$6x = 180^\circ + K \cdot 360^\circ$$

$$x = K \cdot 60^\circ$$

$$x = 30^\circ + K \cdot 60^\circ$$

$$\sin x = 0$$

$$x = 0^\circ, 180^\circ$$

$$0^\circ + K \cdot 360^\circ$$

$$180^\circ + K \cdot 360^\circ$$

$$K=0 \quad 0^\circ, 30^\circ, 180^\circ$$

$$K=1 \quad \cancel{360^\circ}, \cancel{510^\circ}, 60^\circ, 90^\circ$$

$$K=2 \quad 120^\circ, 150^\circ$$

$$K=3 \quad \cancel{180^\circ}, 210^\circ$$

$$K=4 \quad 240^\circ, 270^\circ$$

$$K=5 \quad 300^\circ, 330^\circ$$

Nov 6-10:40 AM

Solve $\sin x - \cos x = \sqrt{2}$

\uparrow \uparrow
 $A=1$ $B=-1$

$$K = \sqrt{A^2 + B^2} = \sqrt{2}$$

Divide by $\sqrt{2}$

$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x = 1$$

$(\cos 315^\circ \sin x + \sin 315^\circ \cos x)$

$$\sin(x + 315^\circ) \quad \cos \alpha = \frac{\sqrt{2}}{2} \quad \sin \alpha = -\frac{\sqrt{2}}{2} \quad \text{Q IV}$$

$$\alpha = \frac{7\pi}{4} = 315^\circ$$

$$\sin(x + 315^\circ) = 1$$

\uparrow
 90°

$$x + 315^\circ = 90^\circ + K \cdot 360^\circ$$

$$x = 90^\circ - 315^\circ + K \cdot 360^\circ$$

$$x = -225^\circ + K \cdot 360^\circ$$

Nov 6-11:09 AM

Graph $y = -\sin 2x + \sqrt{3} \cos 2x$

\uparrow \uparrow
 $A=-1$ $B=\sqrt{3}$

$$K = \sqrt{A^2 + B^2} = \sqrt{4} = 2$$

$$\cos \alpha = \frac{A}{K} \quad \sin \alpha = \frac{B}{K}$$

$$\cos \alpha = \frac{-1}{2} \quad \sin \alpha = \frac{\sqrt{3}}{2}$$

Q II RA $\frac{\pi}{3}$
 $\hookrightarrow \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$2 \left[\cos \frac{2\pi}{3} \sin 2x + \sin \frac{2\pi}{3} \cos 2x \right] = 2 \sin \left(2x + \frac{2\pi}{3} \right)$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

Graph $y = 2 \sin \left(2x + \frac{2\pi}{3} \right)$

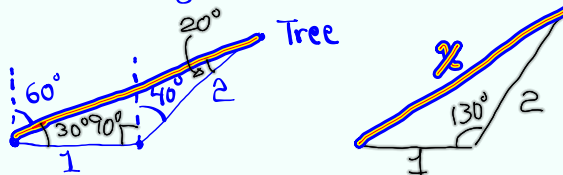
$0 \leq 2x + \frac{2\pi}{3} \leq 2\pi$
 $0 \leq 2x + 2\pi \leq 6\pi$
 $-2\pi \leq 6x \leq 4\pi$
 $-\frac{2\pi}{6} \leq x \leq \frac{4\pi}{6}$
 $-\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$
 $-60^\circ + 120^\circ = 60^\circ / 2 = 30^\circ$

Nov 6-11:15 AM

Diego is 2 miles from a tree with bearing of $N40^\circ E$.

He walks 1 mile west, new bearing is $N60^\circ E$.

How far is Diego from the tree now?



$$x^2 = 1^2 + 2^2 - 2 \cdot 2 \cdot 1 \cdot \cos 130^\circ$$

$$x^2 = 7.571 \quad \boxed{x \approx 2.8 \text{ miles}}$$

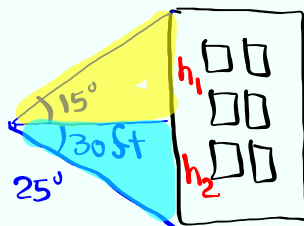
Nov 6-11:26 AM

Angle of elevation to the top of a building is 15°

Angle of depression to the bottom of the building is 25° .

The point is 30 ft from the building.

How tall is the building? $h = h_1 + h_2$



$$\tan 15^\circ = \frac{h_1}{30}$$

$$\tan 25^\circ = \frac{h_2}{30}$$

$$h = 30 \tan 15^\circ +$$

$$30 \tan 25^\circ$$

$$\approx 22 \text{ ft}$$

Nov 6-11:33 AM

$$\sin A = -\frac{5}{8} \quad A \text{ is in Q III}$$

$$\begin{aligned} (-5)^2 + x^2 &= 8^2 \\ 25 + x^2 &= 64 \\ x^2 &= 39 \end{aligned}$$

$$\text{Find } \sin 2A \hat{=} \sin \frac{A}{2}$$

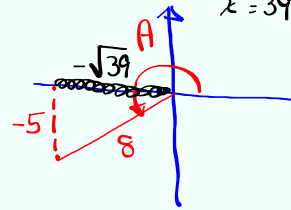
Exact Ans.

$$\sin 2A = 2 \sin A \cos A = 2 \cdot \frac{-5}{8} \cdot \frac{\sqrt{39}}{8} = \frac{5\sqrt{39}}{32}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$= + \sqrt{\frac{1 - \frac{\sqrt{39}}{8}}{2}}$$

$$= \sqrt{\frac{8 + \sqrt{39}}{16}} = \frac{\sqrt{8 + \sqrt{39}}}{4}$$



$$180^\circ < A < 270^\circ$$

$$90^\circ < \frac{A}{2} < 135^\circ$$

Q II

Nov 6-11:38 AM